Product and System Innovation Based on Integrative Design with Ceramic (IDC): Reliability Analysis

In the third chapter on IDC the methods of reliability analysis are addressed. Compared to ductile materials the need of a radically different design of brittle ceramics based on a probabilistic strength calculation arises due to different properties and failure mechanisms. Crucial for the strength of brittle materials under external load are size, shape, density and distribution of microscopically small flaws.

1 Introduction

Over the past 100 years or so, designing with ductile metal materials has become part and parcel of our design engineers' daily routine. Today metal-product development relies on a well-established design methodology and employs a high level of practically proven knowledge of design rules and instructions, whereas designing with ceramic materials particularly for structural applications, quite often ends in disaster.

There is no doubt that a more in-depth study of the specific properties of ceramics is required, because the development of a well-functioning ceramic component is closely linked to a precise knowledge of its mechanical, thermal and chemical properties. Component design and construction must be based on sound material data on one hand, but on the other hand it should not ignore the influence of geometry, loading, and manufacturing, so that characteristic properties of ceramic materials can be used optimally.

Compared to ductile materials, brittle ceramics have a need for a radically different design methodology based on a probabilistic strength calculation due to different properties and failure mechanisms. Because of their susceptibility to cracking even small flaws in brittle materials affect the strength, so that production and machining-related flaws significantly influence the mechanical behavior.

Crucial for the strength of brittle materials under external load are size, shape, density and distribution of microscopically small flaws (cracks). Compared to ductile metals the statistical scatter of the flaws results in a large scatter of the mechanical properties [1]. Therefore, the indication of a failure load is generally not sufficient. Since it is not feasible to determine the size and position of each flaw in a component with reasonable effort and thus predict the strength deterministically, the distribution of flaws is considered in a statistic fracture mechanics framework in order to calculate the survival probability.

For proof of component safety, the required survival probability substitutes the safety factor. The probability of survival (or its complementary measure, failure probability) depends on the stressed volume. This fact must be taken into account when strength parameters, usually determined on standardized specimens, are applied to components. Time-dependent growth of subcritically stressed cracks results in a further reduction of the tolerable stress level. Thus, the strength of a ceramic component decreases with increasing load, component size, and load time.

The key elements of the probabilistic design concept for ceramics are introduced. Furthermore the proposed design concept provides most valuable decision criteria in the IDC framework [1] for selection of load, material, and joining oriented design alternatives.

2 Probabilistic nature of failure in ceramic components

Considering the failure behavior of ceramic components it is generally assumed that fracture originates from natural flaws which grow unstable as soon as a critical load is exceeded. These natural flaws always exist in samples and components primarily due to manufacturing and machining operation, and they widely scatter in size, location and orientation relative to the principal coordinate system. Fracture is triggered by a single unstably propagating crack. That means that the worst flaw in the component determines the tolerable stress. No inter-

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action of flaws among themselves is considered.

2.1 Weibull approach to brittle fracture

The *Weibull* approach to fracture of brittle materials has its fundamentals in the weakest-link theory (WLT). As its name implies, the simplest explanation of the weakest-link phenomenon applies to a chain composed of discrete links. As the chain becomes longer (more links), the probability of encountering a relatively lower strength link increases. Thus, longer chains, on average, have lower strength than shorter chains. To apply this concept to brittle materials like ceramics or glass, the chain links are transposed into volume elements. Thus, the probability of failure increases not only with increase of applied stress but also with increase of the volume of elements at stress. It can be stated that a larger volume of a brittle material will exhibit a lower average strength than a smaller volume of the same material subjected to the same stress state. There is also a flavor of fracture mechanics to this approach in that the underlying assumption is that a ceramic part fails when an inherent flaw is subjected to a critical stress. The flaws in a ceramic part are assumed to be random in size and orientation, resulting in a statistical distribution of the strength. Consider a none-uniform stress distribution $\sigma(x, y, z)$ in a component containing many flaws, and assume that failure occurs due to volume flaws, than the failure probability can be written as

$$P_f = 1 - \exp\left[-\frac{1}{V_0}\int_V \left(\frac{\sigma(x,y,z)}{\sigma_{0V}}\right)^m dV\right] \quad (1)$$

where P_f is the probability of failure, m is the Weibull modulus, σ_{0V} is the Weibull scale parameter, and V_0 is the unit volume. The stress distribution in the component can be expressed as

$$\sigma(x,y,z) = \sigma_p \cdot f(x,y,z) \quad (2)$$

where σ_p represents a stress value which is proportional to component loading, and generally assumed to equal the maximum (peak) stress. For simplification the effective volume V_{eff} is introduced

$$V_{eff} = \int_V f(x,y,z)^m \, dV \quad (3)$$

And substituting this into Eq. (1) yields the resultant fracture probability

$$P_f = 1 - \exp\left[-\frac{V_{eff}}{V_0}\left(\frac{\sigma_p - \sigma_u}{\sigma_{0V}}\right)^m\right] \quad (4)$$

where the subscript V denotes volume dependent terms and σ_u is a threshold value of stress below which no fracture occurs. For ceramics, σ_u is normally taken to be zero. Since there is no generally accepted threshold stress value, this value is convenient while it adds a slight but unknown amount of conservatism. When σ_u is zero, the two-parameter Weibull model is obtained. The scale parameter σ_{0V} then corresponds to the stress level where 63,2 % of specimens with unit volume fail. Both the Weibull modulus m and the Weibull scale parameter may be regarded as material parameters derived from testing. The Weibull modulus m can be thought of as a measure of the variability in the distribution of strength: the lower the Weibull modulus the more variability in the data. The Weibull scale parameter must not be confused with the characteristic strength σ_0, obtained from test specimens, and thus is dependent on geometry and loading type.

Previously only volume flaws have been considered. If fracture occurs due to surface cracks, as it is usually the case, the integration in Eq. (1) has to be performed over a surface instead of volume, and Eq. (4) reads

$$P_f = 1 - \exp\left[-\frac{S_{eff}}{S_0}\left(\frac{\sigma_p - \sigma_u}{\sigma_{0S}}\right)^m\right] \quad (5)$$

where S_{eff} is the effective surface, S_0 the unit surface, and σ_{0S} is the corresponding scale parameter. In the following volume flaws will be considered more closely.

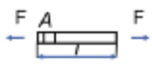
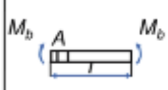
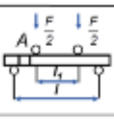
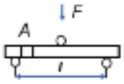
2.2 Size effect

As can be clearly seen from Eq. (4), fracture probability depends on the stress distribution, Weibull modulus, and effective volume, and thus on geometry and size of specimen or component, respectively. By equalizing Eq. (4) for constant fracture probability the size effect is obtained

$$\frac{\sigma_{p,2}}{\sigma_{p,1}}\left(P_f = \text{const}\right) = \left[\frac{V_{eff,1}}{V_{eff,2}}\right]^{\frac{1}{m}} \quad (6)$$

The effect of specimen geometry and Weibull modulus on strength data derived from several test methods is shown in Tab. 1. V is the volume of a tensile specimen (length l, crosssectional area A), which has been used as a scale basis for the effective volume's calculation. For tensile tests, the effective volume is identical to the geometric volume. It can be seen that with increasing Weibull modulus the effective volume decreases, whereas

Tab. 1
Comparison of effective volume and strength data obtained from various test methods

testing method		effective volume V_{eff}				$\frac{\sigma_x}{\sigma_{4PB}} = \left[\frac{V_{eff,4PB}}{V_{eff,x}}\right]^{\frac{1}{m}}$		
	$V = A \cdot l$		*m=10*	*m=15*	*m=20*	*m=10*	*m=15*	*m=20*
tension		V	V	V	V	0.69	0.76	0.80
bending		$V \frac{1}{2(m+1)}$	0.045 V	0.031 V	0.024 V	0.94	0.96	0.97
4 point bending		$V \frac{km+1}{2(m+1)^2}$, $k = \frac{l_1}{l}$	0.025 V	0.017 V	0.012 V	1	1	1
3 point bending		$V \frac{1}{2(m+1)^2}$	0.0041 V	0.0020 V	0.0011 V	1.20	1.15	1.13

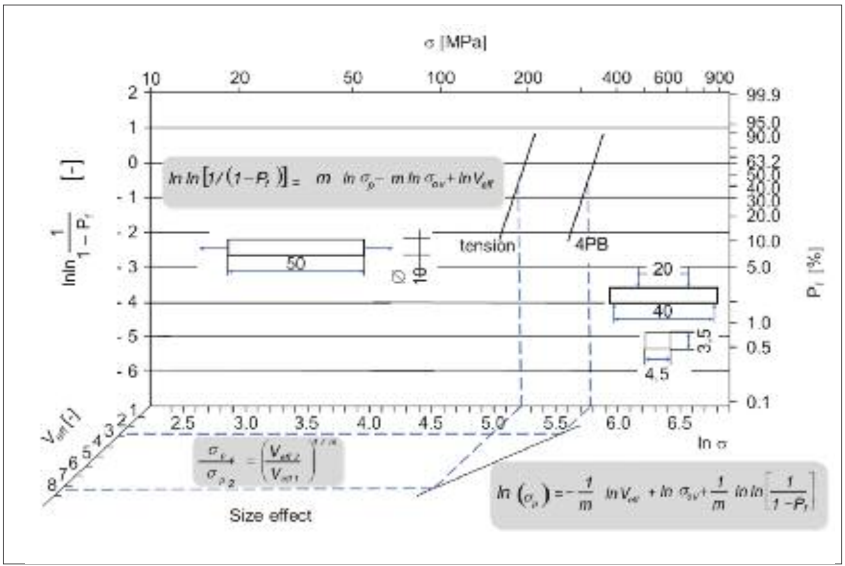


Fig. 1
Relationship between probability of fracture P_f, fracture stress σ, and effective volume V_{eff} obtained from test specimens

strength is decreasing with increasing effective volume.

Attention should be paid to reported values of strength data of ceramics. Usually, flexural or bending strength data derived by applying Weibull statistics to maximum stress at fracture is published. In that case, the scale parameter σ_{0V} which will be applied in the structural analysis has to be determined first.

Fig. 1 shows the relationship between fracture probability, maximum stress at fracture and effective volume of two different Weibull statistics derived from strength data with tensile and four-point-bending test specimens.

2.3 Failure under multiaxial loading

In addition to size and location, the orientation of flaws relative to the principle coordinate system has to be considered. It is again assumed that all orientations are equally likely to occur. The worst combination of defect size, stress, and orientation determines the tolerable load. Classical WLT does not predict failure in a multiaxial stress state and a number of concepts such as the principle of independent action (PIA), Weibull normal stress averaging method, and Batdorf's model have been introduced to expand concepts from a uniaxial to a fully three-dimensional stress state. The PIA method simply transfers the stress tensor to its three principal stresses. Each of them are assumed to act independently. From Eq. (4) it follows that

$$(1-P_f)_V = \exp\left[-\frac{1}{V_0}\sum_{i=1}^{n}\left(\frac{\sigma_I^m(V_i)+\sigma_{II}^m(V_i)+\sigma_{III}^m(V_i)}{\sigma_{OV}^m}\right)V_i\right] \quad (7)$$

where σ_I, σ_{II}, and σ_{III} are the principal stresses. *Batdorf's* model [2] applies fracture mechanics concepts by combining crack geometry and mixed-mode fracture criteria to describe conditions for crack growth.

$$P_{f,V} = 1-\exp\left[-\frac{1}{V_0}\int_V \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left(\frac{\sigma_{eq}}{\sigma_{0V}}\right)^m \sin\alpha\, d\alpha\, d\beta\, dV\right] \quad (8)$$

In Eq. (8) α and β characterize the crack orientation. Further it is assumed that microcracks in the material cause fracture, all cracks are of similar shape, do not interact, and each individual crack has a critical stress.

2.4 Time-dependent fracture

As mentioned earlier the strength of ceramics degrades over time due to various effects, such as oxidation, creep, stress corrosion, and cyclic fatigue. The latter two are representatives of a phenomenon called subcritical crack growth (SCG). SCG initiates on pre-existing flaws. These cracks propagate slowly until they attain a critical load-dependent size, causing unstable propagation. This occurs when the equivalent mode I stress intensity factor K_I equals the fracture toughness K_{Ic}. The SCG failure mechanism is load induced over time and can depend on effects such as chemical reactions with the environment, temperature etc.

A typical v-K_I curve is shown in Fig. 2. In region I crack growth can be expressed by a power law relation

$$v = \frac{da}{dt} = A K_I^n \quad (9)$$

A and *n* are parameters depending on temperature and environment. K_I is given by

$$K_I = \sigma\sqrt{a}\,Y \quad (10)$$

where σ is the stress at the crack tip, *a* the crack size, and *Y* being a geometrical function. After substituting Eq. (10) in Eq. (9), rearrangement and integration provide the time to fracture t_f (lifetime) when a crack of initial size a_i starts to propagate unstably under an applied stress σ. a_i can be directly derived from the so-called inert strength σ_{Ic} (without influence of SCG) and fracture toughness K_{Ic} according to Eq. (10). Finally, one obtains the times to fracture t_s from static tests with constant load σ_s = const. as follows

$$t_s = \mathrm{B}\,\sigma_s^{-n}\,\sigma_{IC}^{n-2} \text{ with } \mathrm{B} = \frac{2}{(n-2)\,\mathrm{A}\,\mathrm{Y}^2\,\mathrm{K}_{IC}^{n-2}} \quad (11)$$

The same approach can be applied for lifetimes obtained from cyclic tests. For further details the reader is referred to [3].

In the following section a brief discussion on how SCG enters fracture probability and lifetime distribution is presented. For reasons of clarity a homogenous uniaxial stress state is assumed. If σ_u is taken to be zero, $\sigma_p = \sigma_{Ic}$, and V_{eff} = 1 mm³, then Eq. (4) reads

$$P_f = 1-\exp\left[-\left(\frac{\sigma_{Ic}}{\sigma_{0V}}\right)^m\right] \quad (12)$$

where *m* and σ_{0V} are the Weibull modulus and scale parameter, respectively. Inert strength σ_{Ic} and time to fracture t_f are both related to the initial flaw size. To obtain the

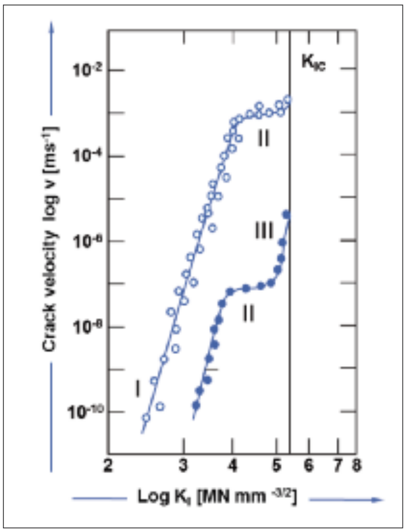


Fig. 2
Typical v-K_I curves

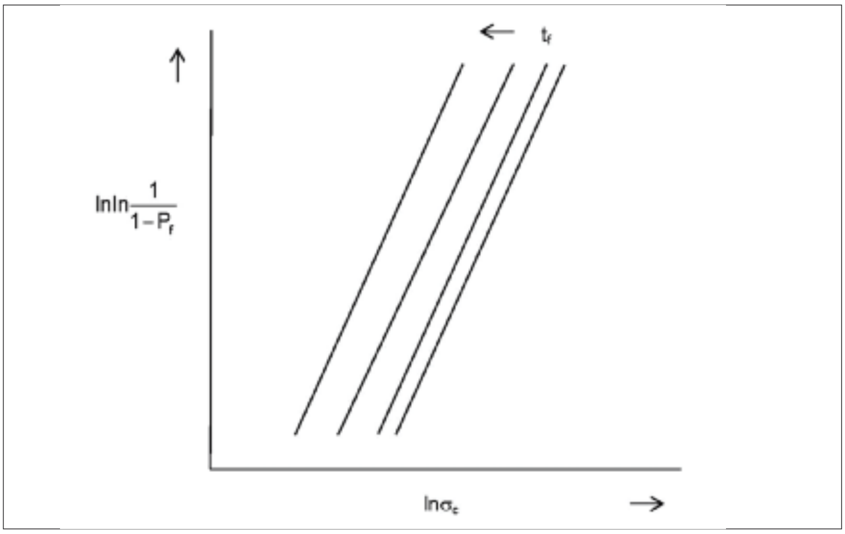


Fig. 3
Schematic representation of a SPT diagram

Weibull distribution of the times to fracture, σ_{Ic} in Eq. (12) is replaced by t_f as given in Eq. (11). This results in the Weibull distribution of lifetimes

$$P_f = 1-\exp\left[-\left(\frac{t_f}{t_0}\right)^{m'}\right] \text{ with } t_0 = B\,\sigma^{-n}\,\sigma_0^{n-2} \quad (13)$$

2.5 Reliability analysis

Calculating component fracture probability requires integration of e.g. Eq. (8). Usually numerical approaches like finite element analysis (FEA) have to be applied for complex geometry, loading, and boundary conditions, respectively. In this case, the component is discretized into a finite number of small elements which are homogeneous and mathematically defined in shape, size, and location by a set of nodes and a shape function. The formulation of the shape function determines the order of the element. From FEA, the stress, strain, and temperature distributions can be obtained. Depending on the failure criterion a full stress tensor at every single *Gauss* point is entered into the component analysis. Therefore, a precise stress distributions over the volume is required. Post-processing codes access the stress tensor in the result data base of FEA software to evaluate the fracture probability $P_{f,i}$ for each single finite element. The resultant probability of fracture of the whole component is the product of the individual probabilities of fracture of all single elements Eq. (14). For the PIA method this results in Eq. (15), and for Batdorf's approach in Eq. (8), respectively.

$$P_{f,tot} = 1-\left[\prod_{i=0}^{n} 1-P_{f,i}\right] \quad (14)$$

$$1-P_{f,tot} = \left[1-P_{f,tot}(\sigma_1)\right]\left[1-P_{f,tot}(\sigma_2)\right]\left[1-P_{f,tot}(\sigma_3)\right] \quad (15)$$

Major tools for structural ceramic component design in today's engineering praxis are knowledge and experience, analytical and numerical stress analysis but also probabilistic reliability analysis. The more inexperienced the design engineer and the more complex the component geometry and/or loading, the more important become the latter two steps. However, in general the following remarks may be helpful:

- Ensure that strength data have been obtained from service-like loading conditions.
- Guarantee that same material grade and surface condition are used for each test
- Increase the accuracy of strength/reliability predictions by increasing the effective volume in mechanical testing (size effect, Tab. 1).
- Consider the size effect when Weibull parameters are determined and applied for component analysis (Eq. 6).
- Use a fast fracture/reliability approach (no SCG) for rough proof of concept (Eqs. 4+5/7+8).
- Apply a time-dependent fracture/reliability approach only when reliable SCG data is given (Eqs. 11, 13).
- First and foremost (to avoid disaster): consult your trusted material supplier/manufacturer in case of any open questions and doubts!

2.6 Strength-Probability-Time (SPT) diagram

An SPT diagram is a graphical representation of the relationship between strength, probability, and time. As an alternative to the concept presented above it allows for estimation of component lifetime as a function of the applied load and required probability of failure. The double logarithmic representation of Eq. (13) yields

$$\ln\ln\frac{1}{1-P_f} = \frac{m\cdot n}{n-2}\ln\sigma_0 + \frac{m}{n-2}\left[\ln t_f - \ln\left(B\sigma_0^{n-2}\right)\right] \quad (16)$$

Fig. 3 shows a schematic SPT diagram. SCG parameters *n* and *A* are obtained from double torsion (DT), double cantilever beam (DCB), or dynamic bending tests, whereas Weibull modulus *m* and strength are derived from bending tests with higher load rates. The plot

$\ln\ln\frac{1}{1-P_f}$ vs. $\ln\sigma_0$ provides a lifetime t_f for each linear with slope $m' = \frac{m\cdot n}{n-2}$

depending on the Weibull modulus *m* and the susceptibility to SCG. As can be seen from Fig. 3, fracture probability is obtained from the applied load and a given lifetime or, vice versa, for a given fracture probability the maximum tolerable stress can be obtained.

3 Conclusion

It's not a surprising new finding that the statistic nature of fracture in ceramics originates from randomly distributed microstructural flaws. It is therefore all the more important that design concepts take into consideration a probabilistic fracture mechanics based approach for strength and reliability analysis. The concept for assessment of fast fracture, time-dependent fracture, and reliability has been known for several decades, but is still not being employed in daily engineering praxis. One reason may be that especially design engineers are educated in a ductile metal world, and as a result persistently apply the learned design methodology. This is not a criticism to the design engineers but rather a serious indictment that existing design methodologies are too rigid and inflexible and moreover that the education of our design engineers at universities proves to be incomplete. It will be certainly not an easy task to change this situation, but how else will we handle future challenges with more demanding requirements for materials and components, arising from diminishing resources, energy saving, and cost and efficiency-increasing measures, other than no longer leaving outstanding characteristic properties of ceramic materials unexploited. This is a call for action and simultaneously a challenge to enhance cooperation between material supplier and manufacturer, design and structural engineers, end-users, and research institutes to provide solutions on how to overcome todays roadblocks. Quite in line with the principle: better well calculated, than boldly guessed!

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